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CORRIGENDUM

Level-set approaches of L_2-type for recovering shape and contrast in ill-posed problems

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The authors are submitting a corrigendum to the above article, published in *Inverse Problems in Science and Engineering* 20(4), 2012, pp. 571–587 [doi: 10.1080/17415977.2011.639452], available at http://dx.doi.org/10.1080/17415977.2011.639452. In what follows, we correct the pcLS approach introduced in our article [1]. The main modifications are: (i) the proof of [1, Lemma 9] is corrected; (ii) the definition of admissible pairs has to be modified accordingly. The convergence analysis results and the numerical experiments presented in the article remain unchanged.

Corrections to the pcLS approach

Differently from [1], we assume that $\phi(x) = i - 1$, $x \in D_i$, $i = 1, 2$, i.e. $\phi(x) \in \{0, 1\}$ a.e. in Ω. Thus, defining the auxiliary functions $\psi_1(t) := 1 - t$ and $\psi_2(t) := t$, the characteristic functions of the subdomains D_i can be written in the form $\chi_{D_i}(x) = \psi_i(\phi(x))$, $i = 1, 2$. Moreover, a solution $u \in X := L_2(\Omega)$ of the operator equation (1) in [1] can be parameterized as follows:

$$u = c^1\psi_1(\phi) + c^2\psi_2(\phi) =: P_{pc}(\phi, c^j). \tag{1}$$

Notice that, the piecewise constant assumption on ϕ corresponds to the constraint $\mathcal{K}(\phi) = 0$, where $\mathcal{K}(\phi) := (\phi)(\phi - 1)$ is a smooth nonlinear operator.

Main changes and corrections

Let $\tilde{D} \subset \Omega$ be an open bounded subset, with the Lebesgue measure $|\tilde{D}| > \gamma > 0$ for a fix γ. We define the following subset of $\mathtt{BV}$:

$$\mathtt{BV}_0(\Omega) := \{\phi \in \mathtt{BV}(\Omega) \,:\, \phi(x) = 0, \text{ a.e. } x \in \tilde{D}\} \tag{2}$$

and redefine the admissible pairs as follows.

Definition 2 (Changed) Let the operators P_{pc} be defined as in (1) and $\tau > 0$. A vector $(\phi, c^j) \in L^2(\Omega) \times \mathbb{R}^2$ is called admissible when $\phi \in \mathtt{BV}_0(\Omega)$ and $|c^2 - c^1| \geq \tau$.

It is worth noticing that, the modification in the definition of the operator $\mathcal{K}$ do not alter the conclusions of [1, Lemma 8].

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LEMMA 9 (Corrected) *Let the operators P_{pc} and $\mathcal{K}$ be defined as above. For $1 \le p < 2$ the following assertions hold true*:

(i) *For every admissible vector (ϕ, c^j) we have $|P_{pc}(\phi, c^j)|_{\mathtt{BV}} \ge \tau |\phi|_{\mathtt{BV}}$.*
(ii) $\mathtt{BV}_0(\Omega)$ *is a closed subset of* $\mathtt{BV}(\Omega)$ *with respect to the $L_p(\Omega)$ convergence. In other words, if $\phi_k \in \mathtt{BV}_0(\Omega)$ is a sequence converging to $\phi \in \mathtt{BV}(\Omega)$ w.r.t. the $L_p(\Omega)$-topology, then $\phi \in \mathtt{BV}_0(\Omega)$.*
(iii) *For every admissible vector (ϕ, c^j), there exist a constant $c > 0$ such that $|P_{pc}(\phi, c^j)|_{\mathtt{BV}} \ge c\, \|\phi\|_{L_2(\Omega)}$.*
(iv) *The functional $\|\mathcal{K}(\cdot)\|_{L_1(\Omega)}$ is weak lower semi-continuous.*

Proof The proof of assertion (i) does not need any corrections. Assertion (ii) follows from the inequality

$$\left(\int_{\tilde{D}} |\phi|^p \,\mathrm{d}x\right)^{\frac{1}{p}} \le \left(\int_{\tilde{D}} |\phi - \phi_k|^p \,\mathrm{d}x\right)^{\frac{1}{p}} + \left(\int_{\tilde{D}} |\phi_k|^p \,\mathrm{d}x\right)^{\frac{1}{p}} \le \|\phi - \phi_k\|_{L_p(\Omega)}$$

(to obtain this inequality one uses the Minkowski inequality). Assertion (iii) follows from assertion (i) and the Poincaré inequality for $\mathtt{BV}$ functions [2, Theorem 1 (ii), p. 189]. To verify the assertion (iv), notice that the equation $\mathcal{K}(t) = 0$ is equivalent to $\tilde{\mathcal{K}}(t) = \frac{1}{4}$, where $\tilde{\mathcal{K}}(t) := \mathcal{K}(t) + \frac{1}{4}$. Thus, it is enough to prove that the functional $\|\tilde{\mathcal{K}}(\cdot)\|_{L_1(\Omega)}$ is weakly l.s.c. Since the real function $t \mapsto \tilde{\mathcal{K}}(t)$ is convex, this property follows from [3, Theorem 1.1, p. 7; and subsequent remark, p. 8]. ■

This lemma is enough to guarantee weak lower semi-continuity of the functional $\|\mathcal{K}(\cdot)\|_{L_1}$. Notice that Lemma 9 (corrected) provides the essential tools needed to derive the main convergence analysis results for the pcLS approach in [1, Theorems 10 and 11].

References

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