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CONSTRUCTAL DESIGN OF NON-UNIFORM X-SHAPED CAVITY

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Abstract. *This paper applies constructal design to study a non-uniform X-shaped cavity that penetrates a conductive solid wall. The goal is to minimize the maximal dimensionless excess of temperature between the solid body and cavity. There is a uniform heat generation on the solid body. The total volume and the volume of the cavity are fixed, but the angle formed between the stems of the cavity may vary. The cavity surfaces are isothermal while the solid body has adiabatic conditions in the outer surface. Results indicate that the optimal X-cavity performs 60.1% better than the C-shaped cavity and 44% better than the T-shaped cavity. However, it has a performance approximately 38% inferior than the performance of the optimized H-shaped cavity.*

Keywords: *Constructal design, enhanced heat transfer, heat conduction, high thermal conductivity*

1. NOMENCLATURE

A	area, m²
D	thickness, m
k	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
L	length, m
q	heat current, W
q'''	heat generated per unit volume, $W \cdot m^{-3}$
T	temperature, K
V	volume, m³
x,y	spatial coordinates, m
W	width, m

Greek symbols

α	angle between the inferior branch of the X and the x axis
β	angle between the superior branch of the X and the x axis
θ	dimensionless temperature, eq. (7)
ϕ	area fraction

Subscripts

max	maximal
min	once minimized
mm	twice minimized
opt	optimal
oo	twice optimized
~	dimensionless variables, Eq. (5 – 9), (11 – 13), (15)

2. INTRODUCTION

Theory Constructal is the thinking that the generation of flow configurations is a physics phenomenon that can be based on a physics principle: The Constructal law of design and evolution. The constructal law states that for a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that it provides easier access to the currents that flow through it (Bejan, 2000, Bejan and Lorente, 2008, Bejan and Zane, 2012, Bejan and Lorente, 2013).

Constructal Law has been used frequently to study complex structures in engineering and nature. Some examples of engineering applications can be seen in the recent literature (Hajmohammadi *et al.*, 2013; Lorenzini *et al.*, 2013,

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Ghaedamini *et al.*, 2011, Chen *et al.*, 2011, Eslami and Jafarpur, 2012). Bejan and Lorente (2013) have also shown a detailed review of literature in the Constructal field.

The importance of fins and cavities (spaces between fins) has attracted the attention of researchers to enhance heat transfer removal from solid bodies where there is heat generation. Recently, constructal design has been applied to improve the performance of isothermal and convective cavities (see Biserni *et al.*, 2004, Biserni *et al.*, 2007, Lorenzini and Rocha, 2009, Lorenzini *et al.*, 2012a, Lorenzini *et al.*, 2012b, Rocha *et al.*, 2007 and Rocha *et al.*, 2010).

The present work continues the body of work on the improvement of the performance of cavities. It applies constructal design to obtain better and better geometries that facilitates the heat flow in a solid body that generates uniform internal heat generation per unit of volume. The body is cooled by isothermal X-shaped cavity. The outer surfaces of the body are adiabatic. The α and β angles formed between the stem of the cavities (see Fig. 1) may vary: they are the two degrees of freedom studied in the present work. The objective function is the excess of temperature between the maximal temperature of the solid body and the cavity which must be minimized subjected to the constraints: total volume and volume of the cavity.

3. MATHEMATICAL MODEL

Consider the conducting body shown in Fig. 1. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with the length L of the total volume. The solid body generates heat uniformly at the volumetric rate q''' (W/m^3) and it is cooled by an X-shaped isothermal cavity. The outer surfaces of the solid are perfectly insulated.

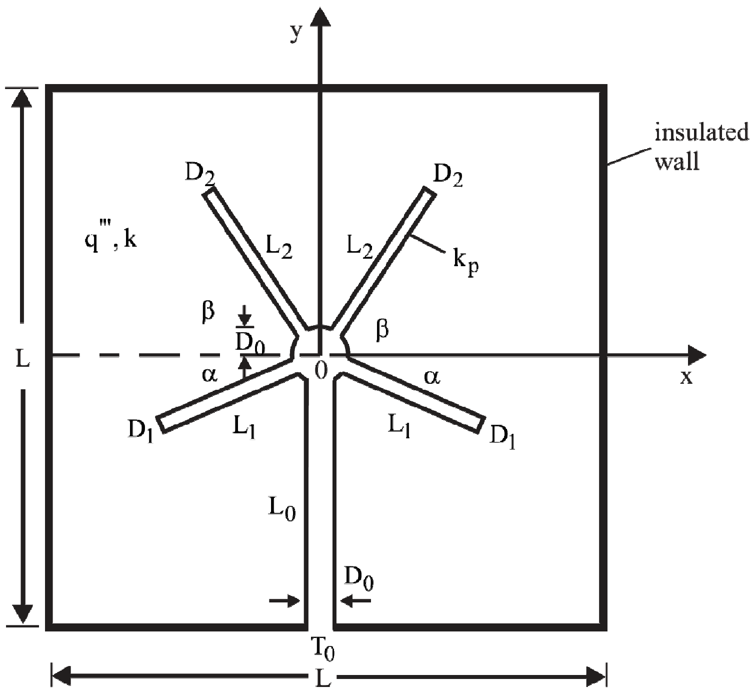

Figure 1. X-shaped isothermal cavity inserted into a solid body

The objective of the analysis is to determine the optimal geometry (α and β defined in Fig. 1) that is calculated by minimizing the dimensionless maximal excess of temperature $(T_{max} – T_0)/(q''' A/k)$.

According to constructal design, this optimization can be subjected to two constraints. The first constraint is the total area,

$$A = L^2 \tag{1}$$

and the second constraint is the approximate area occupied by the X-shaped cavity

$$A_c \cong \pi D_0^2 + D_0 L_0 + 2D_1 L_1 + 2D_2 L_2 \tag{2}$$

Equations (1) - (2) can be expressed as the area fraction

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$$\phi = \frac{A_c}{A} \tag{3}$$

Due to pure observation it was noted that another geometric constraint emerged and it could be approximated by

$$\frac{L}{2} = L_0 + D_0 \, cos(\pi/6) \tag{4}$$

The analysis that delivers the maximal excess of temperature as a function of the geometry consists of solving numerically the dimensionless steady heat conduction with heat generation equation along the solid body,

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \tag{5}$$

where the dimensionless variables are given by

$$\theta = \frac{T - T_0}{q'''A/k}, \tag{6}$$

and

$$\tilde{x}, \tilde{y}, \tilde{L}, \tilde{L}_0, \tilde{L}_1, \tilde{L}_2, \tilde{D}_0, \tilde{D}_1, \tilde{D}_2 = \frac{x, y, L, L_0, L_1, L_2, D_0, D_1, D_2}{L} \tag{7}$$

The outer surfaces are insulated and their boundary conditions are

$$\frac{\partial\theta}{\partial\tilde{n}} = 0 \tag{8}$$

while the cavity surfaces are isothermal and their boundary conditions are

$$\theta_0 = 0 \tag{9}$$

The dimensionless form of equations (1) and (3) and (4) are

$$1 = \tilde{L}^2 \tag{10}$$

$$\phi \cong \pi \, \tilde{D}_0^2 + \tilde{D}_0 \, \tilde{L}_0 + 2\tilde{D}_1 \, \tilde{L}_1 + 2\tilde{D}_2 \, \tilde{L}_2 \tag{11}$$

$$\frac{\tilde{L}}{2} = \tilde{L}_0 + \tilde{D}_0 \, cos(\pi/6) \tag{12}$$

The dimensionless maximal excess of temperature, θ_{max}, is our objective function and is defined as

$$\theta_{max} = \frac{T_{max} - T_0}{q'''A/k} \tag{13}$$

4. NUMERICAL MODEL

The function defined by Eq. (13) can be determined numerically, by solving Eq. (5) for the temperature field in every assumed configuration (α, β), and calculating θ_{max} to see whether θ_{max} can be minimized by varying the configuration. In this sense Eq. (5) is solved using a finite elements code, based on triangular elements, developed in MATLAB environment, precisely the PDE (partial-differential-equations) toolbox (Matlab, 2000). The grid is non-uniform in both $\tilde{x}$ and $\tilde{y}$, and varied from one geometry to the next. The appropriate mesh size was determined by successive refinements, increasing the number of elements four times from the current mesh size to the next mesh size, until the criterion $\left|\left(\theta_{max}^{j} - \theta_{max}^{j+1}\right)/\theta_{max}^{j}\right| < 1\times 10^{-4}$ was satisfied. Table 1 gives an example of how grid independence is achieved. The following results were performed by using a range between 760 and 46.000 triangular elements. The accuracy of the numerical method has already been tested, e.g., by Biserni at al. (2007).

5. OPTIMAL GEOMETRY

The numerical work consisted of determining the temperature field in a large number of configurations of the type shown in Fig.1. In order to understand the effect of each degree of freedom on the geometry configuration and thermal performance, the seek for the optimal shape is performed in two steps. In the first step, we start by simulating the effect of the β angle. Figure 2 shows that there is an optimal β angle that minimizes the dimensionless maximal excess of temperature for several values of α angle when the degrees of freedom L_1/L_0, L_2/L_0, D_1/D_0, D_2/D_0, and ϕ area fraction are fixed. It is also noticed in Fig. 2 that the effect of β over θ_{max} was significantly modified as the value of α is varied. For instance, for α = 0 rad θ_{max} decreases until $\beta_o = 0.47$ and after this value θ_{max} shows only a slight increase. On the opposite, for α = -0.7 rad, it is observed a significant decrease of θ_{max} until the minimal value and, after this point, it is noticed a step increase of θ_{max}. This figure also indicates that there is a second opportunity of optimization (the second step).

Table 1. Numerical tests showing the achievement of grid independence (ϕ = 0.1, $D_1/D_0 = 0.5$, $D_2/D_0 = 0.3$, $L_1/L_0 = 0.8$, $L_2/L_0 = 0.9$, α = - 0.5236[rad], β = 0.7853[rad]).

Number of elements	θ_{max}^{j}	$\left|\left(\theta_{max}^{j} - \theta_{max}^{j+1}\right)/\theta_{max}^{j}\right|$
462	0.0419918	5.8×10^{-4}
1687	0.0425699	2.3×10^{-4}
6429	0.0427985	9.0×10^{-5}
25081	0.0428894	

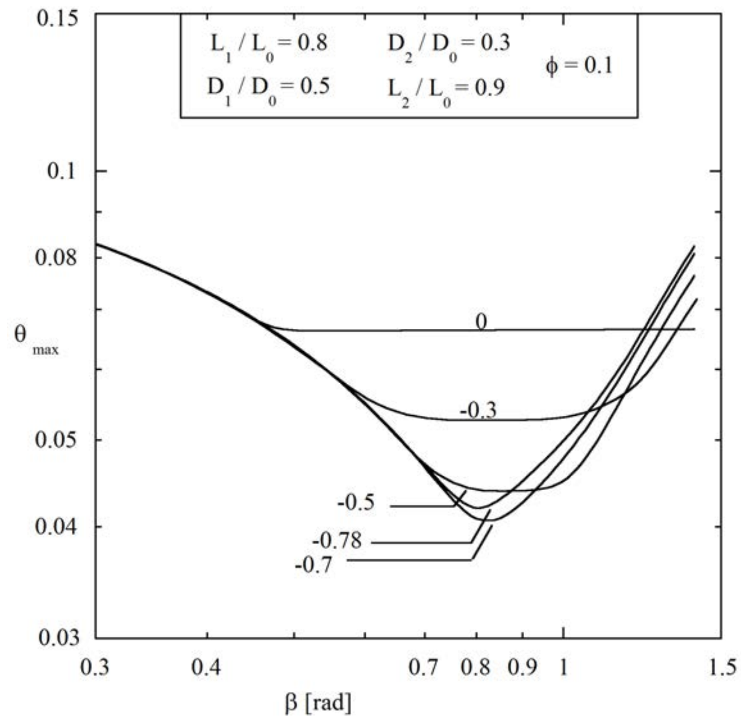

Figure 2. The behaviour of the dimensionless maximal excess of temperature as function of the β angle for several values of the α angle.

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Therefore, the minimal dimensionless maximal excess of temperatures $\theta_{max,m}$ calculated in Fig. 2 are summarized in Fig. 3 as a function of β-angle. This figure shows that the twice minimized $\theta_{max,mm} = 0.03956$ is obtained when the optimal angle α is $\alpha_o = -0.7$.

The once minimized β_o angle as function of the α angle is shown in Fig 4. The β_o is approximately constant and equal 0.79 rad in the range $-0.8 < \beta_o < -0.5$ and decreases linearly to approximately $\beta_o \approx 0.47$ when $\alpha = 0$. Figure 4 also shows that the twice optimized value of β which minimizes two times the maximal excess of temperature is $\beta_{oo} = 0.79$.

The best X-shaped cavity configuration calculated in Fig. 3 and 4 is shown in Fig. 5. This figure shows how the hot spots are better distributed around the outer surfaces of the solid body, therefore the heat can flow easily from the body to the cavity. This result agrees with the optimal distribution of imperfection principle stated by Bejan (2000).

Table 2 shows a comparison among C-, T-, H- and X-cavities under the same conditions. X-cavity performs approximately 60.1% better than the C-shaped cavity and 44% better than the T-shaped cavity. However it has a performance of approximately 38% inferior than the performance of the H-shaped cavity. However, it is important to notice that the H-shaped cavity has been optimized completely (i.e. all its degrees of freedom), while the performance of the X-shaped cavity can be improved optimizing the other degrees of freedom which were kept constant in this study.

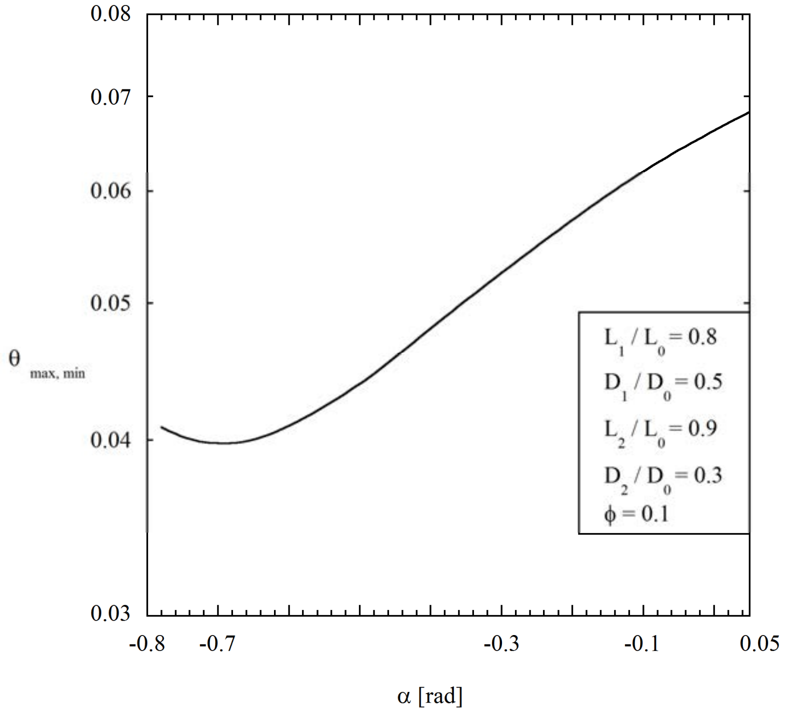


Figure 3. The once minimized dimensionless maximal excess of temperature $\theta_{max,min}$ as function of α angle.

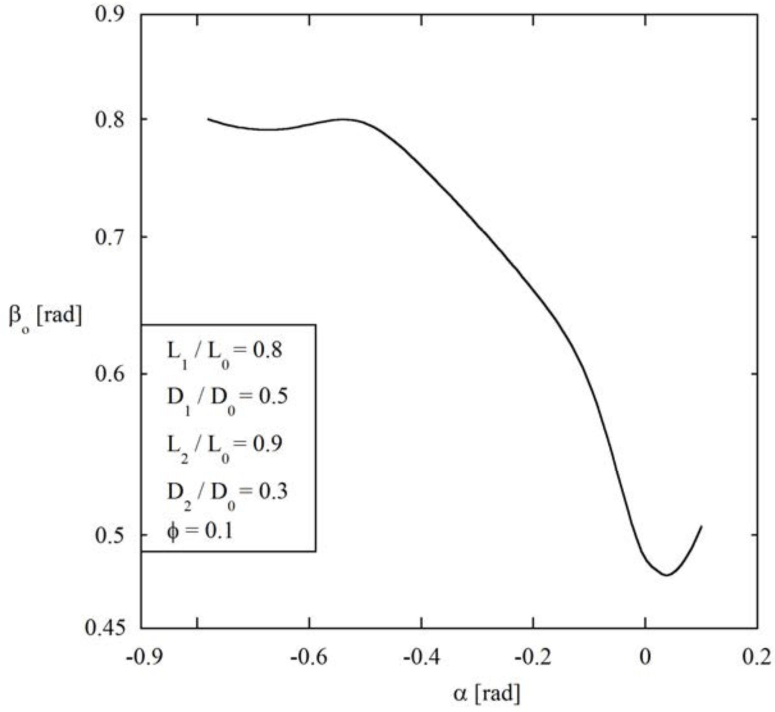
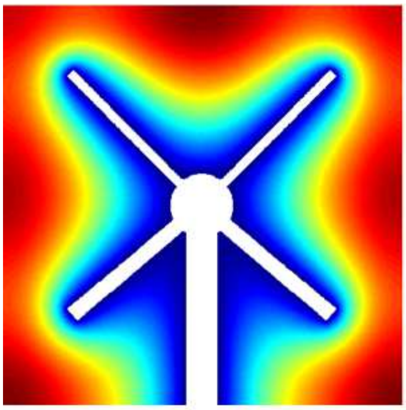


Figure 4. The once optimized β_o as function of α angle.



$$\alpha_o = -0.7 \, [\text{rad}]$$
$$\beta_{oo} = 0.79 \, [\text{rad}]$$
$$\theta_{max,mm} = 0.0395$$

Figure 5. The best shape obtained from Figs. 3 and 4.

Table 2. Comparison of the C-, T- H and X-shaped cavities (H/L = 1 and ϕ = 0.1).

Cavity shape	$\theta_{max,min}$
C-shaped cavity	0.1008
T-shaped cavity	0.0710
H-shaped cavity	0.0245
X-shaped cavity	0.0395

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6. CONCLUSION

This work applied constructal design to search the best architecture that maximizes the performance of non-uniform X-shaped cavity intruding a solid body which generates heat uniformly at the volumetric rate q''' (W/m3). The outer surfaces of the solid were perfectly insulated and the generated heat current was removed by the isothermal X-cavity. In the present work, it was analyzed the minimization the excess admensional maximum temperature in relation to the degrees of freedom and the angles α and β. Results indicated that there is an optimal X-shaped cavity when αo = -0.7 and βoo = 0.79. This ideal form has better distribution of hot spots according to the optimal distribution of the principle of imperfections. Another important finding is that the X-shaped cavity performs approximately 60.1% better than the C-shaped cavity and 44% better than the T-shaped cavity under the same conditions. However, it has a performance of approximately 38% inferior than the performance of the H-shaped cavity. However, the H-shaped cavity has been optimized completely (i.e. all its degrees of freedom), while the performance of the X-shaped cavity can be improved optimizing the other degrees of freedom which were kept constant in this study. The study of the effect of these additional degrees of freedom will be performed in future works.

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